

Triply Periodic Minimal Surfaces Structures in Energy Absorption: A Review of Geometric Topologies, Manufacturing Constraints, and Future Smart Materials

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Abstract: Triply Periodic Minimal Surfaces (TPMS) have emerged as a pivotal class of cellular materials, reshaping lightweight structural design and functional integration in the context of Industry 4.0. This review provides a critical analysis of sheet-based TPMS architectures, benchmarking their performance against conventional strut-based lattices and stochastic metal foams across multiple material systems. The coupled effects of geometric topology (Gyroid, Diamond, Primitive, I-WP, and Lidinoid), additive manufacturing-induced defects, and dynamic energy absorption mechanisms under complex loading conditions are systematically examined. Particular emphasis is placed on quantifying the manufacturing-to-performance gap, demonstrating how surface roughness and lack-of-fusion porosity substantially degrade fatigue resistance and specific energy absorption through localized stress concentrations. Advanced design strategies, including functionally graded materials, numerical homogenization, and multi-phase architectures, are further discussed. The review concludes by outlining future directions toward multifunctional applications and AI-assisted inverse design, providing design-oriented guidelines for selecting flaw-tolerant TPMS architectures in safety-critical engineering systems.

Keywords: Triply Periodic Minimal Surfaces; Energy Absorption; Additive Manufacturing; Selective Laser Melting; Topology Optimization.

1. Introduction and Mathematical Foundations

1.1. Introduction

In the contemporary context of rapid global technological advancement and escalating environmental

regulatory pressures, the imperious demand for lightweight material systems that simultaneously possess superior load-bearing capabilities has become a primary research driver in modern materials science [1, 2]. Particularly within critical high-stake industries such as aerospace, automotive, and defense, structural weight reduction is no longer merely an economic optimization for fuel efficiency; it has evolved into a technical imperative to enhance payload capacity, extend range, and decisively reduce greenhouse gas (GHG) emissions in alignment with Net Zero targets [3, 4]. Over the past half-century, weight reduction efforts have predominantly relied on the substitution of base materials, utilizing light alloys like aluminum (Al), titanium (Ti), or carbon fiber reinforced polymers (CFRP); however, as the physical limits of these bulk materials are asymptotically approached, a fundamental paradigm shift is required—transitioning from “material selection” to “material architecture design” [1, 5]. This conceptual evolution underpins the rapid development of cellular solids and lattice structures, where material distribution is spatially optimized to achieve specific mechanical property combinations unattainable by monolithic natural materials [2, 6]. Among these, Triply Periodic Minimal Surface (TPMS) structures are emerging as a premier candidate to replace traditional honeycombs and stochastic metal foams, offering a unique synergy of continuous geometric topology, high specific surface area, and extensive mechanical customizability [7, 8].

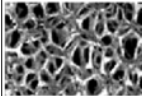
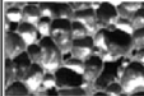


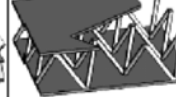

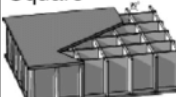



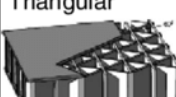



Cellular materials					
Stochastic		Periodic			
Open cell	Closed cell	2D		3D (lattice)	
		Honeycombs	Prismatic	Truss	Textile
		 Hexagonal	 Triangular	 Tetrahedral	 Diamond textile
		 Square	 Diamond	 Pyramidal	 Diamond collinear
		 Triangular	 Navtruss	 3D kagome	 Square textile

Figure 1: Evolution of Cellular Materials. [9]

The transition from stochastic cellular structures to periodic architected materials marks a significant leap in the ability to predict and control mechanical responses within the framework of Industry 4.0 and the digital transformation of manufacturing [10, 11]. Historically, closed-cell aluminum or titanium metal foams were widely adopted as energy-absorbing cores in sandwich panels or automotive crash boxes due to their relatively simple manufacturing processes and low cost. However, the stochastic nature of pore distribution and cell wall thickness in metal foams leads to substantial dispersion in macroscopic mechanical properties, fundamentally complicating precise structural design and necessitating large safety factors that diminish weight reduction efficiency. Conversely, TPMS structures are precisely defined by implicit level-set equations, enabling absolute geometric control at the unit cell level through digital-twin-based design workflows and high-fidelity numerical simulation [12, 13]. This capability

heralds the era of “programmable materials” where engineers can locally tailor wall thickness, cell size, and topological type at any specific voxel within the structure to precisely match the external stress field [14, 15]. Furthermore, the maturation of Additive Manufacturing (AM) technologies—specifically Laser Powder Bed Fusion (LPBF)—over the past decade has acted as a critical enabler, transforming these mathematically complex TPMS designs from theoretical simulations into widespread physical reality that can be integrated into large-scale industrial assemblies for providing a mathematical template that would later prove invaluable to next-generation transportation systems [10, 16]. Although TPMS originate from classical minimal surface theory, their significance in modern materials engineering has been realized primarily through their ability to deliver continuous topology, high surface-to-volume ratio, and predictable mechanical behavior. These attributes, extensively discussed in both fundamental and applied studies, have enabled TPMS to bridge mathematical minimal surface theory and practical engineering design in fields ranging from architected materials to bio-inspired structures [1, 8, 17, 18]. Nature has long utilized TPMS geometries to achieve extraordinary optical and mechanical properties through billions of years of evolutionary refinement, providing a masterclass in structural optimization and energy management [1, 18]. For instance, the photonic crystals found in the Wing scales of the *Callophrys rubi* butterfly are composed of chiral Gyroid networks that manipulate light through Bragg diffraction to produce brilliant green iridescence [17]. Similarly, the exoskeletons of certain beetles and the structural support systems in avian bone marrow demonstrate the implementation of non-zero mean curvature surfaces that maximize fracture toughness while minimizing metabolic cost and weight [18, 19]. By mimicking these natural motifs, modern engineering aims to translate biological efficiency into industrial-scale protection systems that can withstand extreme environments [20]. This review provides a comprehensive dissection of TPMS evolution specific to energy absorption applications, traversing the trajectory from mathematical theoretical foundations through the complexities of additive manufacturing challenges to the intricacies of failure mechanisms under high-strain-rate impact loading [6, 21, 22]. Despite these immense potentials, a significant research gap persists: the lack of systematic, quantitative evaluation concerning the nexus between “Geometric Topology” “Manufacturing Defects” and “Real-world Performance” [23, 24]. This review aims to bridge that divide by synthesizing recent empirical findings to elucidate how manufacturing artifacts like surface roughness and porosity influence the failure modes of TPMS, thereby proposing a “flaw-tolerant” design framework for next-generation energy absorbers that can survive the rigors of industrial deployment and rigorous certification protocols [25, 26].

1.2. Theoretical & Geometric Framework

1.2.1. Fundamental Mathematical Representations and the Weierstrass-Enneper Formalism

To profoundly understand the mechanical behavior of Triply Periodic Minimal Surface (TPMS), one must first grasp their mathematical and topological foundations, which date back to the 19th-century works of Schwarz and Neovius. Triply Periodic Minimal Surfaces are minimal surfaces invariant under a rank-3 lattice of translations, characterized mathematically by a zero mean curvature at every point on the surface ($H = (k_1 + k_2)/2 = 0$), where k_1 and k_2 represent the principal curvatures [21]. This intrinsic property implies that at any point, the surface is locally saddle-shaped or anticlastic, with convex curvature in one principal direction exactly balanced by concave curvature in the orthogonal direction. This endows TPMS with unique geometric features such as exact space partitioning, which facilitates the creation of bicontinuous structures ideal for simultaneous fluid transport or biological tissue ingrowth

where one domain serves as the structural scaffold and the other as the transport network. Furthermore, the complete absence of sharp edges, vertices, or discontinuous nodes—common in strut-based trusses like the Octet-truss or Kelvin cell—eliminates geometric stress concentrators [27]. This ensures that stress flux transmits smoothly throughout the structure via membrane action rather than accumulating at local weak points, which significantly delays the onset of plastic localization and enhances the fatigue life and damage tolerance of the component [23, 28].

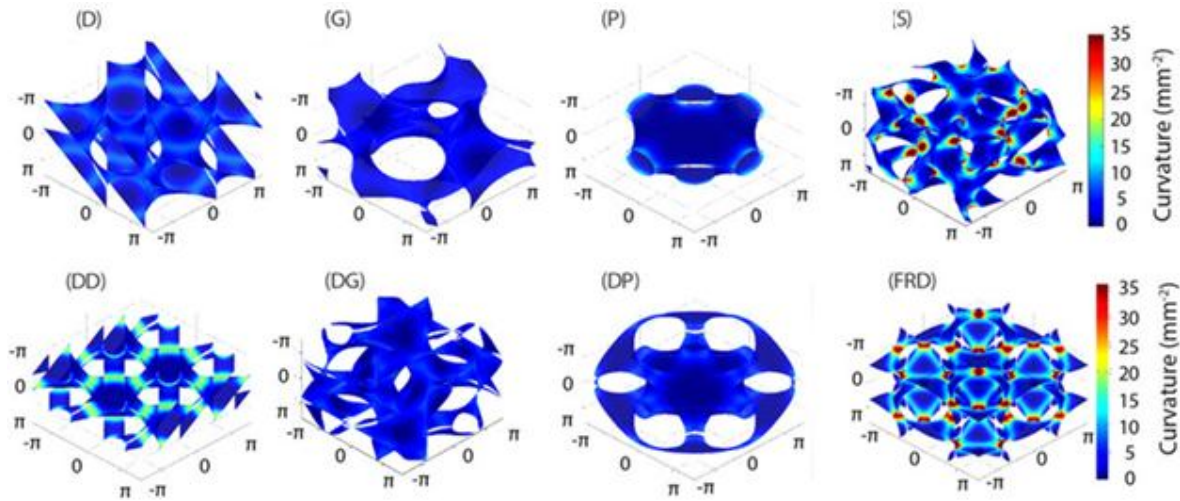


Figure 2: Gaussian curvature $|K|$ distribution along the surface of each TPMS structure. Color bars represent the Gaussian curvature values in mm^{-2} . (a) Diamond (D); (b) Gyroid (G); (c) Schwarz P (P); (d) Fischer–Koch S (S); (e) Double Diamond (DD); (f) Double Gyroid (DG); (g) Double Schwarz P (DP); (h) F-RD. [29]

While TPMS can be rigorously described using the Weierstrass–Enneper formulation, engineering-oriented studies predominantly employ implicit level-set representations due to their numerical efficiency and direct compatibility with additive manufacturing workflows. This approach allows precise control over relative density, wall thickness, and functional grading, facilitating topology optimization and large-scale numerical simulations [8, 12, 13, 15, 27]. For the three fundamental topologies, the governing equations define distinct crystallographic symmetries that dictate their mechanical utility and aesthetic appeal [30]. The Schwarz Primitive (P) surface, following the equation $\cos(x) + \cos(y) + \cos(z) = C$, possesses simple cubic symmetry and is notable for its large, open channels that minimize fluid flow resistance. However, it is mechanically classified as a bending-dominated structure where walls undergo bending rather than stretching, resulting in a lower relative stiffness ($E \propto \rho^2$) but highly stable plastic collapse with minimal stress concentration [31]. In contrast, the Diamond structure, derived from the F-RD diamond crystallographic lattice with $Fd\bar{3}m$ symmetry, possesses high nodal connectivity and tetrapodal coordination, creating a rigid stretching-dominated network where cell walls are primarily loaded in tension or compression [21, 32]. This yields the highest stiffness-to-weight ratio ($E \propto \rho^1$) among the classical triad, though the rigidity often results in brittle-like post-yield softening and large stress oscillations that must be mitigated through density grading or material choice [6, 32]. Occupying the middle ground is the Schoen Gyroid (G) topology ($Ia\bar{3}d$), which is perhaps the most mechanically intriguing as it contains no reflection planes or straight rotational axes. Its primary advantage lies in its exceptional elastic isotropy, with a Zener anisotropy index near unity ($A \approx 1$), making it the premier choice for protective structures where impact vectors are unpredictable, such as in high-performance helmet liners or omni-directional vehicle crash boxes that perform reliably under unpredictable collision

scenarios [13, 21, 28].

1.2.2. Advanced and Novel Topologies: Engineering Specificity and Genus Optimization

Beyond the classical triad, the frontier of Triply Periodic Minimal Surface (TPMS) research is expanding into novel, complex topologies like the I-WP, Lidinoid, and F-RD, which are engineered for highly specialized mechanical functions and multi-load-case performance. The Schoen I-graph-Wrapped Package (I-WP) is defined by a hybrid level-set equation

$$2\cos(x)\cos(y) + 2\cos(y)\cos(z) + 2\cos(z)\cos(x) - (\cos(2x) + \cos(2y) + \cos(2z)) = C$$

and presents a stark behavioral contrast between its solid-network and sheet-based forms [8, 15]. While Solid I-WP tends to form localized shear bands leading to brittle-like collapse, the Sheet-based I-WP exhibits a collective hardening mechanism with superior Specific Energy Absorption (SEA) values ranging from 12-26 MJ/m³, often outperforming the Gyroid in purely axial compression due to the presence of vertical, column-like walls that bear loads efficiently through axial membrane compression and localized folding. Similarly, the rarer but mechanically potent Lidinoid surface has been confirmed by recent Finite Element Analysis (FEA) studies to provide the highest specific stiffness and compressive strength among all TPMS families, surpassing even the Diamond and Neovius structures by optimizing the distribution of wall thickness relative to the local curvature. However, the extreme rigidity of the Lidinoid topology comes at a distinct cost to ductility and energy absorption stability, as these structures—especially when fabricated from brittle or semi-brittle materials like AISi10Mg or PLA composites—tend to undergo catastrophic brittle failure immediately following the yield point, rendering them less suitable for passenger safety applications where gradual deceleration is the primary goal [6, 22, 32]. Instead, Lidinoid is ideal for static structural components requiring minimal deformation under high loads, such as satellite optical benches, precision instrument housings, or deep-sea pressure vessels where structural stability under extreme hydrostatic load is paramount [11, 14]. Other emerging topologies, such as the Split-P surface, are engineered with low elastic modulus specifically to match the stiffness of human trabecular bone to prevent stress shielding in orthopedic implants, while the Fischer-Koch (S) topology is optimized for maximizing surface area and maintaining moderate stiffness in heat exchanger applications [17, 33]. This diversity in topological performance underscores the necessity of a multifaceted design framework that maps specific mechanical requirements to the most appropriate mathematical surface representation. Topological properties such as the Euler characteristic (χ) and the genus (g) play a critical role here; surfaces with a higher genus possess higher connectivity, which generally correlates with improved redundant load paths and flaw tolerance, providing a mathematical metric to evaluate the reliability of an architecture under the presence of stochastic manufacturing-induced defects that are unavoidable in current additive technologies [3, 18, 19].

2. Manufacturing and Computational Modeling

2.1. Manufacturing Considerations and Process-Induced Defects

The practical implementation of Triply Periodic Minimal Surface (TPMS) architectures is inextricably linked to the capabilities and constraints of Additive Manufacturing (AM), specifically Selective Laser Melting (SLM) and Laser Powder Bed Fusion (LPBF), which serve as the primary technological enablers for these complex geometries [10, 11, 16]. The final structural integrity and performance of a printed TPMS part are critically governed by the Volumetric Energy Density (VED), defined as $E_v = P/(v \cdot h \cdot t)$, where P , v , h , and t represent laser power, scanning speed, hatch spacing, and layer thickness, respectively [10, 34]. At the microscopic scale, the laser-matter interaction creates a high-temperature

melt pool whose stability is dictated by a precarious balance between surface tension gradients, known as Marangoni convection, and vapor recoil pressure [11, 16]. Insufficient energy density leads to “lack-of-fusion” porosity, where the molten metal fails to fully wet the underlying layer, leaving behind sharp-edged internal voids that serve as potent failure initiators under cyclic loading [7, 25]. Conversely, excessive energy density can trigger “keyhole” mode welding, where the recoil pressure creates a deep, unstable cavity that traps protective gas bubbles upon solidification, resulting in spherical microporosity that degrades the effective strength of the lattice walls and introduces stochasticity into the mechanical response [25, 26, 34].

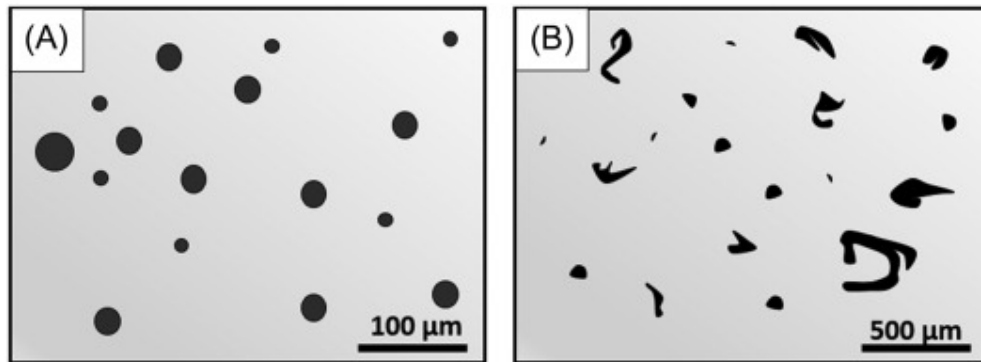


Figure 3: Additive Manufacturing Defects. (a) gas porosity (b) keyhole porosity [35]

Furthermore, the self-supporting capability of TPMS surfaces varies across topologies and is a function of the local overhang angle relative to the build direction, which dictates the need for auxiliary supports. From an industrial perspective, overhang limitations and support removal are not merely geometric constraints but critical barriers to scalability and cost-efficiency. While laboratory-scale studies often neglect these issues, in real manufacturing environments, internal supports that cannot be removed can render certain geometries impractical. In this context, Gyroid and Diamond structures exhibit a distinct advantage due to their self-supporting nature, with most surface angles exceeding the critical threshold (typically 30–45°), allowing fabrication without internal supports. This significantly reduces post-processing requirements and enhances manufacturability at scale.

Table 1. Manufacturability comparison of TPMS topologies in additive manufacturing

Topology	Self-supporting	Support removal	Manufacturability	Industrial suitability
Gyroid	Yes	Not required	High	Excellent
Diamond	Mostly yes	Minimal	High	Excellent
I-WP	Partial	Moderate	Medium	Conditional
Primitive	No	Difficult	Low	Limited

In contrast, Primitive structures and certain low-density TPMS configurations frequently require internal supports due to horizontal features. These supports are often inaccessible for removal, particularly in complex lattice networks, limiting their practical adoption in aerospace and automotive components. Therefore, while multiple TPMS topologies may exhibit comparable mechanical performance under ideal conditions, manufacturability constraints strongly favor self-supporting architectures such as Gyroid and Diamond for industrial deployment. Which significantly limits their practical application in complex internal architectures, as summarized in Table 1. While the Gyroid and Diamond structures generally possess sufficient wall angles—typically exceeding 30 to 45 degrees relative to the build plate to remain printable without auxiliary supports, low-density Primitive structures often exhibit horizontal

overhanging bridges that require internal supports. These internal supports are frequently impossible to remove from the complex labyrinthine channels, thereby limiting the orientation and density range of printable Primitive lattices and increasing the post-processing complexity and total cost. Beyond macro-geometric stability, surface roughness represents a pervasive stochastic defect inherent to powder-bed interactions and the “staircase effect”. In SLM-fabricated TPMS, “down-skin” surfaces (those facing the powder bed) exhibit significantly higher roughness, often with R_a values exceeding $30\mu\text{m}$, compared to some “up-skin” surfaces which maintain values closer to $10 - 15\mu\text{m}$ [23, 26]. This discrepancy is caused by the partial sintering of loose powder particles onto the melt pool boundary a phenomenon exacerbated by the low thermal conductivity of the surrounding powder bed which prevents rapid heat dissipation. These micro-scale irregularities act as severe stress raisers (K_t), initiating premature fatigue cracks and facilitating the localized failure of thin walls under dynamic impact [24-26]. Failure to account for this “effective thickness reduction” in numerical models typically leads to a systematic overestimation of the structural stiffness by as much as 15-25%. Additionally, the extreme thermal gradients often reaching 10^5K/s generated during the rapid heating and cooling cycles of the laser scan lock in substantial residual stresses. These stresses can induce severe warping or even spontaneous delamination of the structure from the build plate, necessitating rigorous post-process stress-relieving heat treatments to restore dimensional accuracy and ductility before the part can be certified for service [10, 26, 34]. From an engineering design perspective, these manufacturing-induced defects should be regarded as intrinsic material characteristics rather than secondary imperfections. Failure to account for surface roughness and porosity typically leads to a systematic overestimation of stiffness and energy absorption by approximately 15–25%. However, this deviation is not universal and depends strongly on material system, relative density, and geometric topology [23, 25, 26].

Table 2. Variation of manufacturing-induced deviation across materials and geometries

Factor	Deviation range
316L (ductile)	10–20%
AlSi10Mg	15–25%
Ti6Al4V (as-built)	25–35%
Low density TPMS	Higher deviation
Gyroid	Lower sensitivity
Primitive / strut	Higher sensitivity

Recent studies indicate that the magnitude of this discrepancy varies significantly across different conditions. For ductile materials such as 316L stainless steel, the deviation is typically within 10–20%, as plastic deformation can partially compensate for geometric imperfections. In contrast, brittle or semi-brittle materials such as Ti6Al4V in as-built condition may exhibit deviations exceeding 25–35% due to early crack initiation at surface defects. Geometric topology also plays a critical role: sheet-based TPMS (e.g., Gyroid) tend to be more defect-tolerant due to their continuous curvature and absence of stress-concentrating nodes, whereas strut-based lattices or sharp-curvature TPMS (e.g., Primitive) exhibit higher sensitivity. Furthermore, the relative density influences defect impact, with low-density structures ($\rho/\rho_s < 0.15$) showing amplified sensitivity due to thinner walls and higher surface-to-volume ratios. Therefore, the commonly cited 15–25% deviation should be interpreted as an average range rather than a universal constant, and defect-aware calibration is required for accurate predictive modeling in engineering applications, depending on material system, topology, and relative density, as summarized in Table 2.

2.2. Numerical Modeling and Computational Fidelity

Accurately predicting the mechanical behavior of Triply Periodic Minimal Surface (TPMS) requires a judicious selection of modeling fidelity that balances computational cost with the required level of physical insight into the deformation mechanics [13, 36]. Discretization strategies span three distinct levels of abstraction: 1D beam elements, 2D shell elements, and 3D solid elements [27, 31]. Beam elements are only suitable for extremely low-density strut-based lattices where nodal connectivity dominates the response, as they fundamentally lack the ability to capture the continuous curvature and membrane stress states characteristic of Sheet-TPMS. Shell elements, such as the S4R quad elements in Abaqus, have emerged as the industry standard for modeling thin-walled Sheet-TPMS due to their ability to efficiently represent membrane stretching and bending with moderate computational burden. However, shell models assume a constant through-thickness profile and thus ignore process-induced defects or roughness, requiring 3D solid elements for high-fidelity failure analysis and crack propagation studies [12, 13]. Solid elements, particularly tetrahedral C3D10M elements, are mandatory when the wall thickness is comparable to the pore size; they further enable the mapping of Computed Tomography (CT) data directly into the FEA mesh, allowing for “Digital Twin” simulations that account for as-built geometric deviations and stochastic porosity that are often simplified in CAD-based models [13, 23, 24] in Table 3.

Table 3. Classification of modeling approaches in TPMS studies based on geometric fidelity

Model type	Geometry source	Defects included	Accuracy	Computational cost
CAD-based	Ideal geometry	None	Low–moderate	Low
Defect-augmented	Modified CAD	Approximate	Moderate	Medium
CT-based	As-built scan	Real defects	High	High

A critical distinction in the current literature lies between studies based on idealized CAD geometries and those incorporating as-built geometries obtained via computed tomography (CT). While CAD-based simulations assume perfectly smooth surfaces and nominal wall thickness, CT-informed models explicitly capture manufacturing-induced defects such as surface roughness, internal porosity, and geometric deviations. As a result, CAD-based studies tend to systematically overestimate stiffness and energy absorption, whereas CT-based simulations provide more realistic predictions that align closely with experimental results. However, CT-based approaches are computationally expensive and therefore less commonly adopted in large-scale parametric studies. To improve clarity, the reviewed literature can be broadly categorized into three levels of fidelity: (i) idealized CAD-based simulations, (ii) defect-augmented numerical models (e.g., reduced thickness or stochastic porosity), and (iii) CT-based digital twin models. Each level represents a trade-off between computational cost and predictive accuracy. To manage the computational complexity of large-scale lattice structures, homogenization methods applied to Representative Volume Elements (RVEs) are typically employed to derive effective material properties for global structural analysis [2, 5, 36]. The conceptual foundation of these methods is the Hill-Mandel condition, which states that the macroscopic energy density must be equal to the volume average of the microscopic energy density within the RVE [13]. More advanced approaches, such as Asymptotic Expansion Homogenization (AEH), utilize a multi-scale expansion of the displacement field to separate the global structural response from the local periodic fluctuations that occur at the unit cell level. The accuracy of these methods depends heavily on the chosen boundary conditions, with Periodic Boundary Conditions (PBC) being the gold standard for predicting bulk properties in infinite lattices, despite being

computationally intensive to implement on unstructured meshes [37]. Simpler alternatives include Kinematic Uniform Boundary Conditions (KUBC) and Static Uniform Boundary Conditions (SUBC), which apply displacements or tractions to the RVE boundaries, respectively [35]. Generally, these conditions bracket the true periodic response, where $KUBC \geq PBC \geq SUBC$ in terms of predicted effective stiffness [5, 13]. For TPMS, employing PBC is essential to capture the correct components of the effective stiffness tensor (C_{ijkl}), ensuring that the simulated response aligns with experimental observations of bulk lattice performance across different loading axes [15, 21, 37]. Shell models are considered the standard method for modeling plate-type TPMS systems due to their high computational efficiency and ability to accurately capture bending-membrane behavior. However, shell models have significant limitations: (i) they do not fully describe micro-surface roughness – a factor directly affecting failure initiation; (ii) they struggle to accurately reflect local thickness variations, especially with 3D-printed TPMS structures; and (iii) they do not capture the three-dimensional crack formation and propagation mechanisms. Therefore, the scope of application of shell models needs to be clearly discussed, and consideration should be given to combining them with solid or multi-scale models to improve accuracy in failure and durability studies. The periodic boundary condition homogenization (PBC) method is often considered the "gold standard" in determining the effective properties of periodic TPMS materials. However, for functionally hierarchical TPMS systems (FG-TPMS), the periodic assumption may no longer be appropriate due to spatial variations in geometry or material. In such cases, applying PBC may lead to inaccuracies in predicting the mechanical response. The authors need to clarify the limitations of PBC and consider alternative methods such as mixed boundary conditions, non-periodic representative domain (RVE) models, or direct full-structure simulation to ensure the reliability of the results. Capturing the non-linear and post-yield response of these structures further necessitates advanced constitutive material models that account for the unique microstructural features of 3D-printed alloys, such as grain refinement, martensitic phases, and texture development [11, 16, 34]. The Johnson-Cook model is frequently utilized for high strain-rate impact simulations, as it explicitly incorporates terms for strain hardening, strain rate sensitivity, and thermal softening that occur during high-intensity loading. While effective for capturing global plastic flow, the Johnson-Cook model is often supplemented with damage evolution models like GISSMO (Generalized Incremental Stress State Model) or the Gurson-Tvergaard-Needleman (GTN) model for porous plasticity and ductile fracture [13, 36]. These models allow for the coupling of fracture strain with the local stress triaxiality and Lode angle parameter, which is critical for predicting the exact onset and propagation of fracture in anisotropic TPMS lattices where stress states are complex and triaxial. For instance, in Sheet-TPMS, failure often originates at the regions of highest negative curvature where stress concentration is maximized; GTN models can accurately simulate the void nucleation and coalescence that precedes such localized failure. This multi-scale modeling approach, combining geometric fidelity with sophisticated damage mechanics, provides the necessary predictive power to design safety-critical TPMS components for aerospace and automotive energy absorption where failure is not an option and protection must be guaranteed [6, 21, 22, 38].

3. Mechanical Performance and Energy Absorption

3.1. Material-Specific Analysis and Architecture-Material Synergy

The ultimate mechanical performance of a Triply Periodic Minimal Surface (TPMS) structure is not merely a function of its geometric topology but is profoundly influenced by its convolution with the intrinsic physical properties of the constituent material. This synergy between architecture and material

determines the dominant failure modes, specifically whether the lattice undergoes stable plastic folding or catastrophic brittle collapse under increasing. For high-performance aerospace applications, Ti6Al4V remains the predominant workhorse due to its exceptional specific strength, reaching approximately 250kNm/kg, and excellent corrosion resistance. However, the extreme cooling rates inherent to Selective Laser Melting ($10^3 - 10^5$ K/s) typically induce a martensitic phase transformation ($\beta \rightarrow \alpha'$), resulting in an acicular microstructure that is prohibitively brittle in its as-built state [10, 34]. Such Ti-TPMS structures frequently fail via adiabatic shear banding, where thermal softening significantly outpaces strain hardening under high-strain conditions, causing localization along 45-degree shear planes and limiting global ductility to less than 10%. To mitigate this, post-process thermal treatments such as annealing or Hot Isostatic Pressing are mandatory to decompose the martensite into a ductile lamellar $\alpha + \beta$ microstructure, thereby enabling the TPMS walls to undergo extensive plastic work and structural folding during an impact event, which is essential for maximizing energy dissipation.

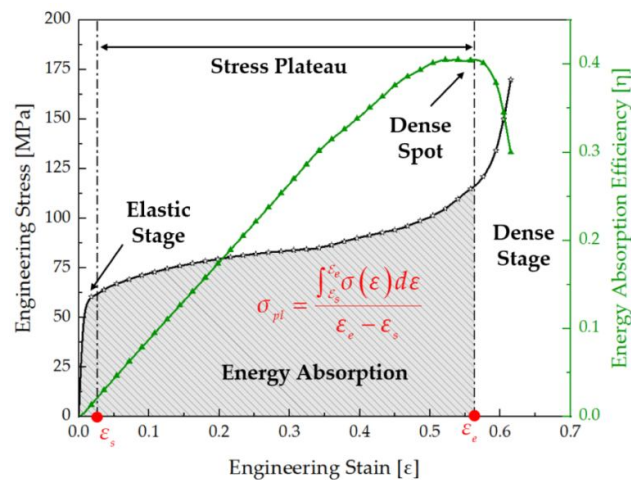


Figure 4: Schematic diagram of the energy absorption of structures. [33]

In the automotive sector, AlSi10Mg and Scalmalloy provide a more cost-effective alternative while maintaining a high stiffness-to-weight ratio and good printability. The microstructure of AlSi10Mg is characterized by a stable cellular network of Si-rich eutectic phases that surround α -Al grains, effectively hindering dislocation movement and preventing the rapid heat localization seen in Titanium alloys. Consequently, Aluminum TPMS lattices exhibit significantly more stable and progressive deformation profiles, where the high thermal conductivity of the alloy ($110 - 170$ W/m · K) ensures that plastic work is distributed uniformly across the entire lattice, delaying the onset of localized failure. For applications requiring maximum energy dissipation under extreme conditions, austenitic stainless steels like 316L are unparalleled due to their extraordinary ductility and the potential for Transformation-Induced Plasticity (TRIP) effects that occur during deformation [6, 33, 35]. These stainless steel lattices demonstrate a uniquely long and stable stress plateau, virtually eliminating brittle fracture and instead allowing the structure to densify into a near-solid block, which is ideal for protective panels designed for blast impulse mitigation, explosion-proof housings, or shock isolation in heavy machinery. At the other end of the material spectrum, 3D-printed polymers such as PA12 or PEEK introduce a viscoelastic dimension, where the mechanical response is strongly strain-rate dependent ($\sigma \propto \dot{\epsilon}^m$) [31]. Furthermore, the integration of short or continuous carbon fibers into these polymeric TPMS structures can enhance the elastic modulus by multiple factors, although it introduces substantial anisotropy and complicates the bonding between the fiber reinforcement and the surrounding matrix, requiring careful optimization of the printing

parameters to avoid interlaminar failure [11, 31, 39].

3.2. Energy Absorption Capabilities and Impact Mechanics

To rigorously quantify the protection efficiency of Triply Periodic Minimal Surface (TPMS) structures, the academic community has established several standardized metrics under ISO 13314, with the most critical being Specific Energy Absorption (SEA) and Crushing Force Efficiency (CFE) [21, 28, 30]. SEA is defined as the total energy dissipated per unit mass up to the onset of densification ($SEA = \frac{1}{\rho^*} \int_0^{\varepsilon_d} \sigma(\varepsilon) d\varepsilon$), where ρ^* represents the apparent density of the lattice [5, 30]. High-performance aluminum Gyroids typically achieve SEA values between 25 – 45 kJ/kg, significantly outperforming stochastic foams and conventional honeycombs of equivalent density through their superior material utilization. Complementing SEA is the Crushing Force Efficiency (CFE), which measures the uniformity of the loading and is vital for limiting the peak deceleration experienced by sensitive payloads or human organs during a crash event [4, 28, 32]. A critical design consideration is the trade-off between SEA and peak force two important indices, Specific Energy Absorption (SEA) and Crushing Force Efficiency (CFE), the trade-off between them in safety applications needs further clarification. In crashworthiness design practice, the goal is not only to maximize energy absorption but also to control peak force to reduce acceleration transmitted to people or equipment. A structure with high SEA but high peak force can cause serious injury, especially in applications such as automobiles or protective equipment. Therefore, it is necessary to emphasize that design optimization must be based on composite indices such as Crashworthiness Index (SEA/Peak Force), while clearly analyzing the trade-off between energy absorption capacity and biosafety. The "dynamic hardening" effect at high strain rates is described as beneficial because it increases plateau stress and overall energy absorption capacity. However, it should be clarified that this phenomenon can also lead to a significant increase in initial peak force, especially during the initial contact phase when stress waves have not yet propagated evenly within the structure. This can be detrimental in systems protecting humans, where minimizing peak acceleration is required. Therefore, the authors should further discuss the balance between the benefits of increased SEA and the risks of increased peak force under dynamic loading, and suggest design strategies such as gradient density distribution, multilayer structures, or hybrid TPMS to effectively control both factors. While Diamond TPMS structures often exhibit a lower CFE (50 – 70) due to the discrete, layer-by-layer collapse of their rigid cells—which generates high-amplitude stress oscillations—Gyroid lattices consistently offer near-ideal CFE values exceeding 90. This superior behavior is facilitated by their continuous, shear-coupled bending paths that allow for a smooth, monotonic plateau stress. Furthermore, the onset of densification strain (ε_d) represents the point at which the pores have completely collapsed and the stress rises exponentially; maximizing the useful stroke of the absorber through delayed densification is a primary goal in energy-absorption design, often achieved by tailoring the wall thickness to be thinner at the impact interface to initiate progressive collapse and maintain a constant deceleration force for the duration of the impact [32, 33, 37]. To strengthen the quantitative comparison between TPMS architectures and conventional cellular materials, a broader benchmarking across different structural families is required. Table 2 summarizes representative ranges of Specific Energy Absorption (SEA), stiffness scaling, and deformation characteristics for TPMS, strut-based lattices, and stochastic foams under comparable relative density conditions ($\rho/\rho_s \approx 0.1-0.3$). It can be observed that sheet-based TPMS consistently outperform stochastic foams by approximately 30–80% in SEA, while also exhibiting significantly lower variability due to their deterministic geometry. Compared to strut-based lattices such

as BCC or Kelvin cells, TPMS structures demonstrate 15–40% higher SEA, primarily due to the activation of membrane stretching rather than bending-dominated deformation.

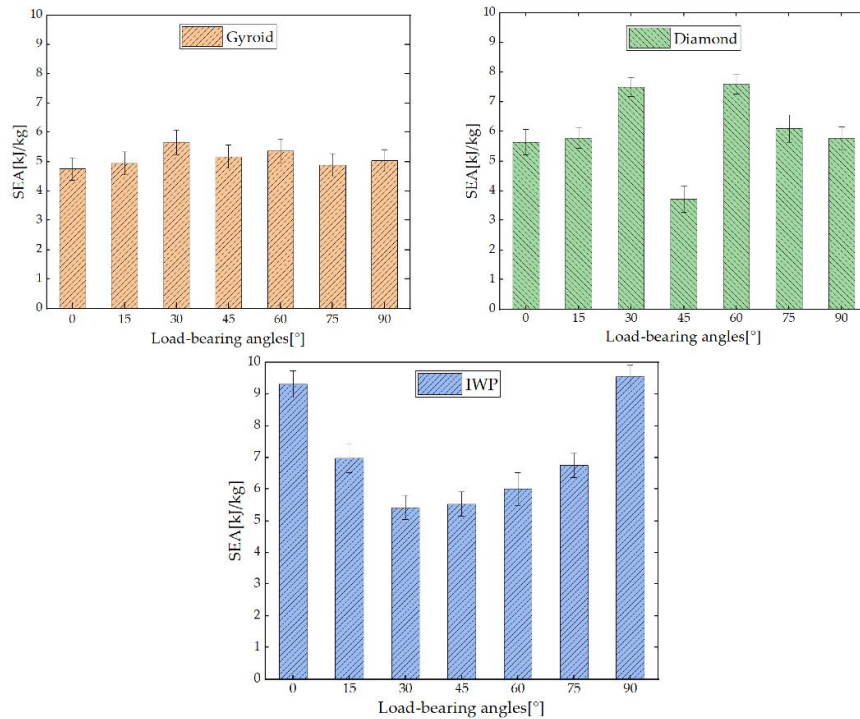


Figure 5: SEA of Gyroid, Diamond and IWP structures at different load-bearing angles [33]

Table 4. Quantitative comparison between TPMS, lattice structures, and stochastic foams

Structure type	Deformation mode	SEA (J/g)	CFE (%)	Variability	Key limitation
TPMS (Gyroid)	Stretching-dominated	15–30	>90	Low	Moderate stiffness
TPMS (Diamond)	Stretching-dominated	10–22	65–80	Low	Stress oscillation
TPMS (I-WP)	Mixed	18–35	>85	Medium	Defect sensitivity
BCC lattice	Bending-dominated	8–18	60–75	Medium	Node failure
Kelvin cell	Bending-dominated	6–15	55–70	Medium	Low stiffness
Metal foam	Random collapse	5–12	50–65	High	Large scatter

Among TPMS families, Gyroid structures show the most stable response with CFE > 90%, while Diamond structures exhibit higher stiffness but increased stress oscillations in Table 4. Sheet-IWP structures achieve the highest SEA values (up to ~35 J/g), albeit with higher sensitivity to defects. These results confirm that the mechanical superiority of TPMS is not only qualitative but can be quantitatively established across multiple performance metrics. Table 5 summarizes the reported specific energy absorption and crushing behavior of commonly investigated TPMS architectures, highlighting the trade-off between energy absorption efficiency and structural stiffness. The mechanical superiority of Sheet-TPMS over traditional strut-based lattices, such as the BCC or Kelvin cell, is

primarily rooted in the activation of membrane stretching deformation modes throughout the entire surface area. In strut-based systems, the mechanical response is dominated by the bending of individual beams, where plastic hinges form exclusively at the nodes, thereby limiting the volume of material contributing to plastic work and reducing the overall efficiency of the absorber [27, 28, 30].

Table 5. Comparative summary of reported energy absorption performance of TPMS-based structures fabricated by additive manufacturing.

TPMS topology	Material	Loading condition	SEA (J/g)	CFE (%)	Key advantage	Limitation	Reference
Gyroid	AlSi10Mg	Quasi-static	15–30	>90	Isotropic plateau	Moderate stiffness	[21, 33]
Diamond	Ti6Al4V	Quasi-static	10–22	65–80	High stiffness	Stress oscillation	[32]
Sheet I-WP	316L	Quasi-static	18–35	>85	High SEA	Defect sensitivity	[33]
Lidinoïd	AlSi10Mg	Quasi-static	8–15	<60	High rigidity	Brittle collapse	[22]

In contrast, the doubly-curved shell elements of a Sheet-TPMS experience in-plane tension and compression, known as membrane stretching, as the surface deforms. Because the energy required to stretch a shell is mathematically proportional to its thickness (t), while bending energy is proportional to t^3 , the Sheet-TPMS architecture absorbs significantly more energy for a given relative density. Comparative studies have demonstrated that although traditional aluminum honeycombs provide exceptional axial energy absorption, their performance catastrophically fails under off-axis or oblique loading due to global buckling of the cell walls [1, 21, 32]. Gyroid structures, possessing cubic symmetry and no straight load paths, maintain a quasi-isotropic yielding behavior, retaining more than 85% of their energy absorption capacity regardless of the impact angle, providing essential direction-tolerance for vehicular crashworthiness where collision vectors are rarely aligned perfectly with the coordinate axes [15, 21, 31]. Under dynamic, high-strain-rate impact conditions, typically exceeding $500s^{-1}$, the mechanical response of TPMS is further governed by the complex interplay of inertia effects and the microstructural strain-rate sensitivity of the base alloy [22, 38]. When a TPMS lattice is subjected to a rapid external impulse, a stress wave propagates through the structure at the local speed of sound in the porous medium [21, 36]. If the impact velocity is comparable to this wave speed, the structure undergoes a transition from a quasi-static homogenized collapse to a cubic-power-law dynamic shock-like collapse. In this regime, deformation is localized at the impact face and moves toward the support as a compaction front, effectively isolating the protected volume from the initial impulse [32, 33]. This dynamic enhancement can increase the plateau stress by up to 50%, a phenomenon termed the “dynamic hardening effect” [38]. For metallic TPMS, this is primarily driven by the microstructural strain-rate sensitivity (e.g., the high rate-sensitivity of 316L stainless steel), while for polymeric lattices, it is dominated by the viscoelastic-viscoplastic transition and adiabatic heating [21, 31]. Advanced numerical studies using the Split-Hopkinson Pressure Bar technique have revealed that the Gyroid topology is particularly adept at attenuating these shock waves due to the high tortuosity of its load paths, which effectively scatters longitudinal stress waves into transverse shear waves, thereby protecting the underlying structure from high-intensity stress pulses that could cause electronics failure or traumatic

injury [22, 27, 40]. The dissipation mechanisms are further diversified by local Hertzian contact mechanics and internal friction as the cell walls fold and touch during the transition to densification [13, 21, 36]. This internal self-contact adds a secondary stiffening mechanism that contributes to the total energy absorption, especially in high-density lattices ($V_f > 0.3$) where the pore size is small compared to the wall thickness. By applying modern finite element contact algorithms within Abaqus/Explicit, researchers have identified that the specific friction coefficient between the printed surfaces significantly influences the post-yield stability and the total energy absorbed; higher friction increases the apparent plateau stress but can also trigger premature ligament fracture in brittle materials like Ti6Al4V [13, 24, 25]. Consequently, the optimization of TPMS for energy absorption requires a holistic consideration of geometric topology, material ductility, and the manufacturing-induced surface roughness, which modulates the local contact behavior [6, 7, 21, 26]. This multifaceted approach is allowing for the design of “flaw-tolerant” architectures that maintain more than 90% of their theoretical performance even when widespread printing defects are present, a critical requirement for safety-critical aerospace and defense components that perform reliably under the most demanding conditions.

4. Advanced Design and Multifunctionality

4.1. Multifunctional Applications and Functional Densification

The true transformative power of Triply Periodic Minimal Surface (TPMS) lies in its capacity for “Functional Densification”—the ability to perform multiple physical roles simultaneously within a single material volume, effectively turning the structural component into a functional system [14, 41]. In the domain of thermal management, the exceptionally high specific surface areas of thin-walled Shell-TPMS, often exceeding $2000\text{m}^2/\text{m}^3$, facilitate their use as high-efficiency heat exchangers and cold plates for high-power electronics [15, 41]. The tortuous, interconnected channels of Gyroid and Diamond lattices induce complex mixing and Dean vortices that disrupt the thermal boundary layer, resulting in Nusselt numbers that are 2-4 times higher than those achieved by straight micro-channels for equivalent pumping power. This multifunctionality is demonstrated in EV battery cooling systems, where a TPMS chassis can serve as both a structural crash-absorbing frame and an active liquid cooling jacket, effectively increasing the system’s energy density and reducing the total part count [4, 41]. By integrating the structural requirements of crash protection with the thermal requirements of heat dissipation, engineers can eliminate redundant components, leading to a “systems-level” weight reduction that far exceeds what is possible through individual component optimization alone. This multifunctional logic extends into the biomedical field, where TPMS geometries mimic the morphology of natural trabecular bone more closely than any truss-based alternative, providing an ideal substrate for bone tissue engineering and orthopedic implants. The continuous topological connectivity of these surfaces ensures high permeability for oxygen and nutrient transport, which is essential for maintaining cell viability in large-scale tissue engineering scaffolds and promoting rapid healing [17-19]. By tailoring the intrinsic permeability of the scaffold through the manipulation of the level-set isovalue, researchers can accelerate osteointegration while simultaneously matching the elastic modulus of the surrounding bone to prevent stress shielding—a common cause of implant failure and bone resorption [14, 17, 18]. Furthermore, the complex connectivity of TPMS scatters and dissipates acoustic energy via multiple reflections and viscous losses within the narrow labyrinths [40, 41]. By integrating stiff metal matrices with soft viscoelastic fillers or air voids, researchers can create local resonance and phononic bandgaps that attenuate vibration and noise across broad frequency ranges, opening new possibilities for structural aerospace

panels that are concurrently insulating, sound-proofing, and load-bearing. This ability to add the multi-physics response of a TPMS at the unit cell level is the cornerstone of the next generation of smart, architected materials that blur the lines between material physics and structural engineering, creating structures that respond to their environment [42].

4.2. Advanced Design Strategies and Topological Optimization

Moving beyond uniform density designs, modern research exploits the spatial design freedom of Triply Periodic Minimal Surface (TPMS) via Functionally Graded Materials (FGM), where the relative density is no longer constant but spatially distinct ($\rho(x, y, z)$) [15, 27, 37]. This biomimetic approach, inspired by the hierarchical density gradients in bamboo or bone, typically utilizes a sigmoid function to prevent stress concentrations at abrupt interfaces. These gradients follow the mathematical form

$$V_f(z) = V_{\min} + (V_{\max} - V_{\min}) / (1 + e^{-k(z-z_0)})$$

where the parameter k controls the gradient steepness. Such a soft-hard-soft profile is often optimal for vehicular crashworthiness, as it provides a low initial peak face—minimizing the deceleration pulse—followed by a high-energy absorbing core that prevents cabin intrusion and protects occupants. The objective of these graded designs is to maximize the Crashworthiness Index (C_{crash}), defined as the ratio of Specific Energy Absorption to the Peak Force [28, 29, 32]. Quantitative studies have confirmed that FGM-TPMS architectures can boost energy absorption by up to 60% over uniform counterparts while tailoring the force-displacement curve to reach a theoretically ideal rectangular shape, which maximizes the stroke efficiency of the absorber within the finite spatial constraints of a vehicle's crumple zone [6, 29, 31].

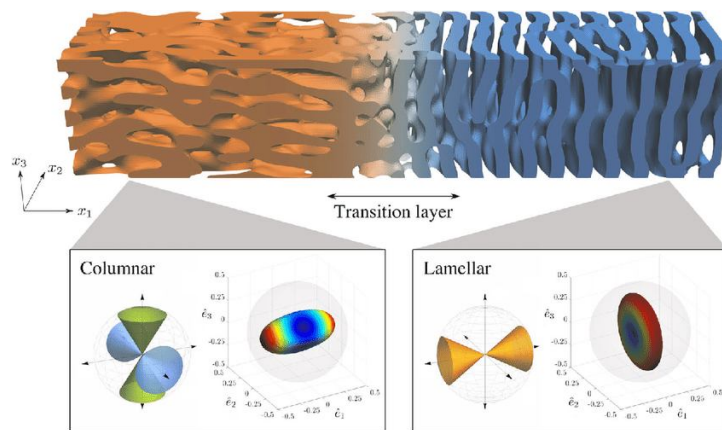


Figure 6: Functional grading. A spatially variant architecture interpolating between columnar and lamellar topologies [38]

Another frontier in topological design is the creation of Hybrid or Heterogeneous Morphologies, which combine distinct TPMS families, such as a ductile Gyroid core with a rigid Diamond skin, to exploit the synergistic benefits of both topologies within a single part. The primary mathematical challenge in these hybrid systems is joining the disparate topologies without creating geometric discontinuities that act as failure initiators. Advanced modeling frameworks frequently utilize the Cahn-Hilliard equation or Sigmoid weighting functions to generate smooth transition regions, where the level-set field is interpolated as $\phi_{\text{hybrid}} = (1 - w)\phi_G + w\phi_D$ [13, 15]. This approach creates a continuous morphing lattice that occupies a unique mechanical property space, effectively eliminating interface delamination a common failure mode in traditional multi-material or bonded composites. Furthermore, the integration of Topology Optimization (TO) with TPMS elevates design to the level of algorithmic automation [42-

44]. TO algorithms, such as the SIMP method, compute the optimal density field required to support specific load cases, which is then mapped directly to the local isovalue of the TPMS. Generative design studies of helmet liners, for instance, have shown that mapping the Von Mises stress distribution from impact tests directly to a Gyroid density gradient can reduce the Head Injury Criterion score by more than 30% compared to conventional isotropic foams, providing superior protection against traumatic brain injury [32, 42]. Finalizing the architectural discussion is the concept of Bio-inspired Hierarchical Design, which implements structural hierarchy by creating “lattices within lattices” to push the limits of material efficiency and damage tolerance. By fabricating the thick walls of a macro-scale TPMS structure from smaller, micro-scale TPMS lattices, researchers introduce secondary and tertiary energy dissipation mechanisms such as multi-scale buckling and crack deflection. These include ligament blunting and the arrest of crack tips, which significantly increase the fracture toughness (K_{IC}) of the overall structure. This fractal-like architecture exhibits a remarkable “flaw tolerance,” maintaining its structural integrity and energy absorption capacity even in the presence of printing defects or local damage. Such hierarchical meta-materials are evolving from academic curiosities into robust engineering solutions for safety-critical systems, blurring the lines between material physics and structural engineering to create structures that are both incredibly light and resilient [14, 18, 19, 25].

4.3. Design Guidelines for Energy Absorbing Triply Periodic Minimal Surface Structures

Based on the reviewed literature, several design guidelines can be extracted. Sheet-based Gyroid structures fabricated from ductile alloys such as AlSi₁₀Mg or 316L provide stable and isotropic energy absorption and are therefore suitable for impact mitigation applications [21, 33]. When stiffness is prioritized over energy absorption, Diamond or Lidinoid topologies may be selected, although density grading is often required to mitigate stress oscillations and brittle collapse [22, 32]. Importantly, defect-aware numerical calibration remains essential to ensure reliable performance under dynamic loading conditions [25].

5. Future Perspectives and Conclusion

5.1. In-Situ Characterization and Dynamic Testing Protocols

To validate these theoretical frameworks, the integration of advanced In-Situ Characterization techniques is becoming mandatory for verifying the predictive accuracy of multi-scale models and understanding real-time failure mechanics [13, 21, 22]. Synchronized high-speed photography ($> 10^5$ fps) and Digital Image Correlation (DIC) allow for the real-time mapping of strain fields across the Triply Periodic Minimal Surface (TPMS) surface during impact tests, revealing the localized formation of shear bands and plastic hinges. These maps provide critical data for calibrating damage mechanics models and identifying the onset of sub-surface fracture which is often hidden from view. Furthermore, the use of Synchrotron X-ray Micro-Tomography during quasi-static compression enables researchers to visualize the internal pore collapse and wall-to-wall contact mechanics with micron-level resolution in three dimensions. These 4D (3D + time) datasets are revolutionizing our understanding of “dynamic hardening,” demonstrating that internal self-contact is not merely a terminal event but a progressive stiffening mechanism that contributes up to 30% of the total energy dissipated in high-density lattices [21, 22, 40]. Standardizing these testing protocols under high-strain-rate regimes, using equipment like the Hopkinson Pressure Bar, will be essential for the certification of TPMS components in mission-critical aerospace protection systems where experimental validation is a mandatory prerequisite for flight safety.

5.2. Scaling Laws and Gibson-Ashby Relations for Triply Periodic Minimal Surface

A cornerstone of cellular material science is the application of Scaling Laws, which relate the macroscopic mechanical properties to the relative density (ρ^*/ρ_s) through analytical power-law correlations. For Triply Periodic Minimal Surface (TPMS), the traditional Gibson-Ashby relations must be refined to account for the sheet-based vs. solid-network morphologies and the influence of Gaussian curvature. The effective Young's modulus follows the relation $E^*/E_s = C_1(\rho^*/\rho_s)^n$, where the exponent n ranges from 1.0 (stretching-dominated) to 2.0 (bending-dominated) [5, 30]. Deep technical analysis reveals that Diamond TPMS typically exhibits $n \approx 1.1$, signifying near-ideal structural efficiency and making it the optimal choice for high-stiffness applications where mass is at a premium, while Primitive TPMS shows $n \approx 1.9$, closer to the behavior of stochastic foams but with much higher predictability and reliability. Furthermore, the yield strength follows $\sigma^*/\sigma_s = C_2(\rho^*/\rho_s)^p$. By meticulously calibrating the constants C_1 and C_2 through massive parametric FEA sweeps across different topologies and material systems, researchers are establishing "Topological Maps" that guide material selection in a data-driven manner. These maps allow engineers to instantly identify the optimal TPMS type and density for a specific load-stroke requirement, significantly accelerating the design-to-deployment cycle for automotive crash boxes and aerospace impact attenuators where time-to-market is critical [14, 21, 32, 36, 42, 45].

5.3. Future Directions: 4D Printing and AI-Driven Design

The convergence of smart materials and 4D Printing heralds a new era of adaptive, stimulus-responsive Triply Periodic Minimal Surface (TPMS) structures that can alter their topology, stiffness, or shape in real-time [38, 42]. These advanced TPMS architectures, typically fabricated from Shape Memory Alloys like NiTi or Shape Memory Polymers, utilize the reversible solid-state phase transformation. A 4D-printed TPMS bumper, for example, could remain in a soft state for pedestrian safety but instantly stiffen upon detecting a high-speed collision. Furthermore, these smart TPMS lattices possess self-healing capabilities. This shape-memory functionality is particularly well-suited to the continuous topology of TPMS [14, 46]. In parallel, Artificial Intelligence and Machine Learning are becoming indispensable partners in the design of next-generation TPMS. The immense complexity of the multifunctional design space often exceeds human intuition. Deep Learning surrogate models can now predict complex mechanical attributes in milliseconds with accuracies exceeding 95%. Furthermore, Physics-Informed Neural Networks (PINNs) are being developed to integrate physical conservation laws. This ensures that the AI-generated TPMS designs strictly obey the laws of continuum mechanics, leading to Inverse Design workflows [42, 44]. However, the path to widespread industrial adoption is constrained by the persistent challenges of Sustainability, Scalability, and Life Cycle Management [11, 39, 48]. Research is shifting toward the use of recycled metal powders and the development of bio-composites. TPMS structures contribute to sustainability efforts by minimizing the total mass of raw material, but the complexities of recycling these intricate lattice geometries remain a significant hurdle. Comprehensive Life Cycle Assessments (LCA) are therefore essential. Moreover, scaling TPMS to industrial dimensions requires high-productivity methods like Wire Arc Additive Manufacturing (WAAM). Establishing international testing standards and modular joining strategies will be the final step in bridging the gap between theoretical research and safety-critical industrial certification [10, 11, 14]. Although functionally hierarchical TPMS (FG-TPMS) and hybrid structures are presented as very promising directions, the level of evidence supporting the relationship between experimental and numerical simulations needs to be clarified. Currently, most results on performance optimization (SEA, strength, strain stability) are still

primarily based on FEA simulations or homogenization models, while experimental validation is limited due to fabrication costs and geometric complexity. This distinction needs to be clearly stated to avoid overestimating the technology readiness level, while emphasizing the urgent need for large-scale experimental studies to validate numerical predictions.

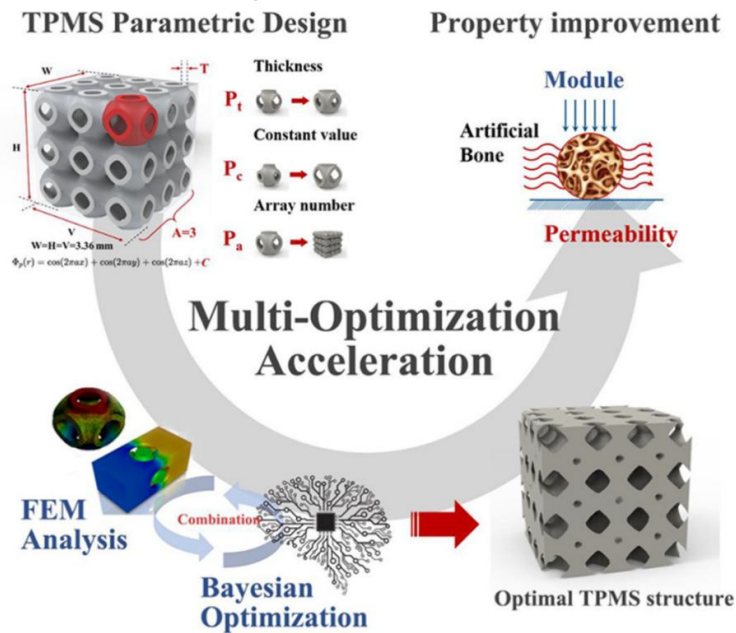


Figure 7: AI and Machine Learning in TPMS Design [47].

The concepts of multi-level hierarchical TPMS and 4D printing are future-oriented, however, many technical barriers still exist. In particular, the resolution limitations of fabrication technologies (e.g., LPBF) make it difficult to realize multi-level structures with very small feature dimensions, leading to significant geometric deviations from the design. Furthermore, smart materials such as shape-memory alloys (SMA) or shape-memory polymers face long-term durability issues, especially fatigue due to repeated phase transition cycles. Therefore, a brief discussion of these limitations is needed to balance theoretical potential and practical applicability. AI- and machine learning-based TPMS design is mentioned as a powerful tool for inverse design optimization; however, the training data source needs clarification. Most current models are trained on simulation-driven datasets, which assume ideal geometry and do not consider fabrication defects. This can lead to model bias, where optimized designs are no longer suitable when transitioning to experimental conditions. Most current models are trained on simulation-driven datasets, as well as methods such as physics-informed neural networks (PINNs) to minimize bias and improve the generalizability of AI models.

6. Conclusion

The ascent of Triply Periodic Minimal Surfaces from mathematical curiosities to high-performance structural components represents a significant milestone in additive manufacturing and material design history. By synthesizing continuous geometric topology with the spatial design freedom of 3D printing, Triply Periodic Minimal Surface (TPMS) architectures offer a unique suite of mechanical and physical properties that are unattainable with traditional cellular solids, honeycombs, or stochastic foams. This review has systematically explored the nexus between geometric topology, manufacturing constraints, and dynamic energy absorption, highlighting the Gyroid and Diamond structures as premier candidates for vehicular protection, orthopedic stabilization, and thermal management. While the challenges of

surface roughness, residual stresses, and computational complexity persist, the emergence of functionally graded designs, AI-driven inverse design, and stimulus-responsive smart materials provides a clear and robust roadmap for overcoming these hurdles in the next decade. As industrial sectors continue to prioritize lightweighting, sustainability, and multi-functional performance, TPMS will undoubtedly play a pivotal role in the engineering of next-generation, high-performance systems for aerospace, defense, and beyond, fundamentally redefining our relationship with structured matter and the limits of engineering possibility. Overall, this review highlights that the performance of TPMS energy absorbers is governed not solely by topology but by the coupled effects of geometry, material ductility, and manufacturing-induced defects. Future research should prioritize defect-aware design methodologies, standardized high-strain-rate testing, and data-driven optimization to facilitate the transition of TPMS from laboratory-scale concepts to certifiable engineering solutions (Zhang et al., 2018; Tancogne-Dejean and Mohr, 2018; Novak et al., 2023).

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